

# A unique $\mathbb{Z}_4^R$ symmetry for the MSSM

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We consider the possible anomaly free Abelian discrete symmetries of the MSSM that forbid the  $\mu$ -term at perturbative order. Allowing for anomaly cancellation via the Green-Schwarz mechanism we identify discrete  $R$ -symmetries as the only possibility and prove that there is a unique  $\mathbb{Z}_4^R$  symmetry that commutes with  $\text{SO}(10)$ . We argue that non-perturbative effects will generate a  $\mu$ -term of electroweak order thus solving the  $\mu$ -problem. The non-perturbative effects break the  $\mathbb{Z}_4^R$  symmetry leaving an exact  $\mathbb{Z}_2$  matter parity. As a result dimension four baryon- and lepton-number violating operators are absent while, at the non-perturbative level, dimension five baryon- and lepton-number violating operators get induced but are highly suppressed so that the nucleon decay rate is well within present bounds.

## I. INTRODUCTION

Supersymmetric extensions of the Standard Model (SM) are very popular as they can solve the hierarchy problem, stabilizing the electroweak scale against a high scale associated with new physics. The simplest such extension, the MSSM, assumes the minimal number of new particle states. However there are several problems that immediately arise in its construction associated with terms in the Lagrangian that are allowed by supersymmetry and by the gauge symmetry of the SM. At dimension three there is a Higgs mass term with coefficient  $\mu$  (as well as  $R$  parity violating terms  $\mu_i H L_i$ ), that, if unsuppressed, reintroduces the hierarchy problem – the so-called  $\mu$ -problem. At dimension four and five there are baryon- and lepton-number violating terms that must be strongly suppressed to prevent unacceptably fast nucleon decay.

Discrete symmetries play an important role in controlling these terms. It is normally assumed that the MSSM should also be invariant under a  $\mathbb{Z}_2$  matter-parity [1–3], that distinguishes between lepton and down-type Higgs doublets. Such discrete symmetries may be constrained by the requirement of anomaly freedom [4–6]. This is certainly the case if they come from a spontaneously broken gauge symmetry. It is also the case if they come from a string theory; for instance, in orbifolds they can arise as discrete remnants of the Lorentz group in the compact space. It has also been argued that non-gauge discrete

symmetries are violated by gravitational effects rendering them ineffective [4].

In this paper we revisit the question what is the underlying symmetry of the MSSM, focusing on anomaly free discrete symmetries to avoid the appearance of new light gauge degrees of freedom. The symmetry should be capable of ensuring that the nucleon is stable, at least within present bounds. Early attempts, assuming the MSSM spectrum, classified low-order anomaly free Abelian discrete symmetries, including  $R$ -symmetries, that can stabilize the nucleon. Matter parity is anomaly free but allows dimension five nucleon decay operators [7]. A  $\mathbb{Z}_3$  “baryon-triality” is anomaly free that forbids both dimension four baryon number violation operators and dimension five nucleon decay operators but allows lepton number violating dimension four operators. The combination of these two gives a  $\mathbb{Z}_6$  “proton hexality” symmetry [8] that forbids all dimension four baryon and lepton number violating operators and dimension five nucleon decay operators. However these symmetries allow the problematic  $\mu$  term; indeed it was a requirement that it should be allowed.

Here we adopt a different philosophy and look for anomaly free discrete symmetries that forbid both the  $\mu$  term and all dimension four and five baryon and lepton number violating operators. Of course an electroweak scale  $\mu$  term is needed but we argue that it should arise through spontaneous non-perturbative breaking of the discrete symmetry thus solving the  $\mu$ -problem. We also require that, unlike baryon-triality or proton hexality (cf.

[9]), the discrete symmetry should commute with a simple Grand Unified group such as  $SU(5)$  or  $SO(10)$ , thus readily preserving the attractive features of Grand or string unification. In addition we require that all fermion masses, including neutrino masses, should be allowed by the symmetry.

Remarkably we find that in the  $SO(10)$  case there is a unique solution, a  $\mathbb{Z}_4^R$   $R$ -symmetry. The symmetry may be broken non-perturbatively generating a  $\mu$  term of the correct order plus dimension five baryon and lepton number violating operators that are sufficiently suppressed to be consistent with bounds on nucleon decay. A  $\mathbb{Z}_2$  symmetry is left unbroken, equivalent to matter parity, that forbids the generation of dimension four baryon and lepton number violating terms and ensures the LSP is stable. The spontaneous breaking of the discrete symmetry leads to a potential domain wall problem but we show this is avoided provided a relatively mild constraint on the reheat temperature after inflation is satisfied.

Anomaly cancellation proceeds via the Green-Schwarz (GS) mechanism [7, 10]. In recent years it has become clear that anomaly free discrete symmetries of this type need not originate from the so-called ‘anomalous  $U(1)$ ’ but can have a string origin [11, 12] although, at least in heterotic orbifolds, there is a tight relation between these symmetries and the anomalous  $U(1)$  [12]. The GS anomaly cancellation requires that the anomaly coefficients be universal. Because these symmetries arise from string compactifications, the effects that violate them are well under control, and they can be viewed as approximate symmetries.

The paper is organized as follows. We first show that only discrete  $R$  symmetries can satisfy the constraints discussed above. Then we prove that, for the case the symmetry commutes with  $SO(10)$ , there is a unique discrete  $\mathbb{Z}_4^R$  symmetry which allows for the usual Yukawa couplings and the Weinberg operator generating Majorana masses for the neutrinos; anomaly cancellation proceeds via the Green-Schwarz mechanism. We then consider the phenomenological implications of the model, including the cosmological implications. Finally we comment on the possible origin of this  $\mathbb{Z}_4^R$  symmetry and briefly discuss an explicit string-derived model with the exact MSSM spectrum below the string scale with this symmetry.

## II. A UNIQUE $\mathbb{Z}_4^R$ SYMMETRY FOR THE MSSM

In this section we prove that with the minimal field content of the MSSM there is a unique discrete symmetry with the following features:

- (i) Anomaly cancellation (allowing for a GS term).
- (ii) Consistency with  $SO(10)$ .
- (iii) No  $\mu$  term at the perturbative level.

- (iv) Quark, charged lepton and neutrino masses allowed.

Consider a  $\mathbb{Z}_N$  symmetry under which the matter superfields have charge  $q^{(f)}$ . For the case  $\mathbb{Z}_N$  is an  $R$ -symmetry the fermion components have charge  $q^{(f)} - 1$  under the convention that the superpotential  $\mathcal{W}$  has  $\mathbb{Z}_N^R$  charge 2. In general, the anomaly coefficients are given by

$$A_{G-G-\mathbb{Z}_N} = \sum_{\mathbf{r}^{(f)}} \ell(\mathbf{r}^{(f)}) (q^{(f)} - R) + \ell(\text{adj}) \cdot R, \quad (1)$$

where  $G$  is the gauge group,  $q^{(f)}$  denotes the chiral superfield  $\mathbb{Z}_N$  charge and  $\ell(\text{adj})$  is the Dynkin index of the adjoint representation of  $G$ . For an  $R$ -symmetry  $R = 1$ , otherwise  $R = 0$ . The sum runs over the irreducible representations  $\mathbf{r}^{(f)}$  of  $G$  of the chiral fields with Dynkin index  $\ell(\mathbf{r}^{(f)})$ . Our conventions are such that  $\ell(\mathbf{M}) = 1/2$  for  $SU(M)$  and  $\ell(\mathbf{M}) = 1$  for  $SO(M)$ . The condition of anomaly cancellation corresponds to

$$A_{G-G-\mathbb{Z}_N} = \rho \pmod{\eta}, \quad (2)$$

where

$$\eta := \begin{cases} N & \text{for } N \text{ odd} , \\ N/2 & \text{for } N \text{ even} . \end{cases} \quad (3)$$

In the absence of a GS term the constant  $\rho = 0$ . Allowing for a GS term the anomaly cancellation condition is relaxed to the condition  $A_{G-G-\mathbb{Z}_N} = \rho$  for the (suitably normalised) gauge factors of the SM.

We now impose the condition that  $\mathbb{Z}_N$  should commute with  $SU(5)$  and assign discrete charges  $q_{\mathbf{10}_i}$  and  $q_{\mathbf{\bar{5}}_i}$  to the  $\mathbf{10}$  and  $\mathbf{\bar{5}}$  representations making up family  $i$ . The coefficients for the mixed  $SU(3)_C$  and  $SU(2)_L$  anomalies are

$$A_{SU(3)-SU(3)-\mathbb{Z}_N} = \frac{1}{2} \sum_i [3 \cdot q_{\mathbf{10}_i} + q_{\mathbf{\bar{5}}_i} - 4R] + 3R, \quad (4)$$

$$A_{SU(2)-SU(2)-\mathbb{Z}_N} = \frac{1}{2} \sum_i [3 \cdot q_{\mathbf{10}_i} + q_{\mathbf{\bar{5}}_i} - 4R] + 2R + \frac{1}{2} (q_H + q_{\bar{H}} - 2R), \quad (5)$$

where  $q_H$  and  $q_{\bar{H}}$  denote the  $\mathbb{Z}_N$  charges of the up-type and down-type Higgs doublets,  $H$  and  $\bar{H}$  respectively.

Allowing for the GS term, anomaly cancellation (universality) requires (cf. [13])

$$(q_H + q_{\bar{H}}) = 4R \pmod{2\eta}. \quad (6)$$

This should be compared to the condition that a Higgs mass term is allowed which is

$$(q_H + q_{\bar{H}}) = 2R \pmod{N}. \quad (7)$$

We immediately see that for a non- $R$  symmetry ( $R = 0$ ) the requirement that  $\mathbb{Z}_N$  commute with  $SU(5)$  means

that the Higgs mass term in the superpotential is allowed and the  $\mu$ -problem remains (cf. the similar discussion in [14]).

The situation is different for the case of an  $R$  symmetry ( $R = 1$ ) and it is not difficult to demonstrate that there are solutions to eq. (6) that do not satisfy eq. (7) and thus solve the  $\mu$ -problem. In fact one can show that any solution that forbids the dimension five nucleon decay operators also forbids the  $\mu$ -term and moreover that  $N$  should be a divisor of 24. We will discuss the general case elsewhere [15] but here we show that in the special case that  $\mathbb{Z}_N$  commutes with  $\text{SO}(10)$  there is a unique solution to the anomaly cancellation equations and that it does solve the  $\mu$ -problem. In this case  $q_{10_i} = q_{\bar{5}_i} = q$  where, to allow for interfamily mixing, the charges must be family independent.

To generate masses for the quarks, charged leptons and neutrinos we require that the  $u$ - and  $d$ -type Yukawa couplings and the Weinberg operator  $(LH)^2$  be allowed.<sup>1</sup> This yields the following conditions between the  $R$ -charges:

$$2q + q_H = 2 \pmod{N}, \quad (8)$$

$$2q + q_{\bar{H}} = 2 \pmod{N}, \quad (9)$$

$$2q + 2q_H = 2 \pmod{N} \quad (10)$$

with solution

$$q_H = q_{\bar{H}} = 0 \pmod{N}, \quad (11)$$

$$2q = 2 \pmod{N}. \quad (12)$$

Inserting this in eq. (6) shows that  $N$  can take two values only,  $N = 2$  or  $N = 4$ . The  $\mathbb{Z}_2^R$  symmetry does not forbid the  $\mu$  term and indeed there are no meaningful discrete  $R$  symmetries of order 2 (cf. e.g. [16] for a recent discussion). We are therefore left with the unique possibility of a  $\mathbb{Z}_4^R$  symmetry with  $\mathbb{Z}_4^R$  charges as given in table I. This symmetry has been considered before in [17] using the Giudice-Masiero mechanism [18] to generate the  $\mu$  term. Another version of this symmetry with  $\rho = 0$  and extra matter has been discussed in [19].

$Q$	$U^c$	$E^c$	$D^c$	$L$	$H$	$\bar{H}$
1	1	1	1	1	0	0

TABLE I:  $\mathbb{Z}_4^R$  charge assignment for the MSSM superfields.

The last step is to check the remaining anomaly cancellation conditions. Mixed  $\text{U}(1)$ - $\text{U}(1)$ - $\mathbb{Z}_N$  anomalies are often ignored as they do not give meaningful constraints unless one knows the normalization of the charges [8, 10]. However, in the case of hypercharge  $Y$  the normalization

is fixed by the underlying GUT. The resulting  $\text{U}(1)_Y$ - $\text{U}(1)_Y$ - $\mathbb{Z}_N^R$  anomaly condition is

$$\begin{aligned} A_{\text{U}(1)_Y\text{-U}(1)_Y\text{-}\mathbb{Z}_N^R} &= \frac{3}{5} \cdot \left\{ 2 \cdot \left(\frac{1}{2}\right)^2 [q_H + q_{\bar{H}} - 2] \right. \\ &\quad + 3(q_{\bar{5}} - 1) \left[ 2 \left(\frac{1}{2}\right)^2 + 3 \left(\frac{1}{3}\right)^2 \right] \\ &\quad \left. + 3(q_{10} - 1) \left[ 6 \left(\frac{1}{6}\right)^2 + 3 \left(\frac{2}{3}\right)^2 + (1)^2 \right] \right\} \\ &= \rho \pmod{\eta}. \end{aligned} \quad (13)$$

Note that this anomaly coefficient is not invariant under shifting some discrete charges by multiples of  $N$ . That is, there are equivalent  $\mathbb{Z}_N^R$  charge assignments, leading to different anomaly coefficients. We find that the true anomaly constraint is that there has to exist a charge assignment under which the conditions of anomaly freedom or anomaly universality are satisfied. For the case of  $\mathbb{Z}_4^R$   $\rho = 1$  and  $\eta = 2$ . In this case, one may check that the anomaly cancellation condition, eq. (13), is satisfied for the choice that both Higgs superfields have  $R$  charge  $-4$  rather than 0.

The  $\mathbb{Z}_N^3$  anomaly does not yield model independent constraints [6, 12]; a more detailed discussion will be presented in [15]. However there is one further anomaly cancellation condition of interest, namely the grav-grav- $\mathbb{Z}_N^R$  graviton anomaly. It too is often ignored as it can always be satisfied by adding SM singlet fields but it is still of some interest because the existence of additional *light* singlet states is potentially of phenomenological interest. We have

$$\begin{aligned} A_{\text{grav-grav-}\mathbb{Z}_N^R} &= -21 + 8 + 3 + 1 \\ &\quad + 3 \{ 10 \cdot (q_{10} - 1) + 5 (q_{\bar{5}} - 1) \} \\ &\quad + 2 (q_H + q_{\bar{H}} - 2) = 24\rho \pmod{\eta}. \end{aligned} \quad (14)$$

For the case of  $\mathbb{Z}_4^R$  this constraint is  $-9 - 4 = 24 \pmod{2}$  which is not satisfied. Thus there must be additional SM singlet state(s) with  $\mathbb{Z}_4^R$  charges  $s_i$  such that  $\sum_i (s_i - 1)$  is odd. We will discuss what mass they may acquire shortly.

### III. $\mathbb{Z}_4^R$ PHENOMENOLOGY

Clearly, the charge assignment given in table I is consistent with Grand Unification for matter. In  $\text{SO}(10)$  language it corresponds to giving the **16**-plet a  $\mathbb{Z}_4^R$  charge 1, such that the matter fermions transform trivially, and  $\mathbb{Z}_4^R$  charge  $-4 \hat{=} 0$  to the Higgs fields contained in the **10**-plet. Notice that successful doublet-triplet splitting is required for these anomalies to be universal, *i.e.* the  $\mathbb{Z}_4^R$  does not commute with  $\text{SO}(10)$  in the Higgs sector. This is the usual doublet-triplet splitting problem that is most elegantly solved in string unification via Wilson line breaking.

The structure of the renormalisable terms of the  $\mathbb{Z}_4^R$  model is identical to that of the usual MSSM with matter parity with the exception that the  $\mu$  term is absent. The

<sup>1</sup> Allowing the Weinberg operator is equivalent to adding right-handed neutrinos  $\nu^c$  with charge 1. Then Yukawa couplings of  $\nu^c$  and Majorana mass terms are also permitted.

terms of dimension five differ in that the baryon- and lepton-number violating terms  $QQQL$  and  $U^c U^c D^c E^c$  are absent. However the dimension five Weinberg operator  $(LH)^2$  is allowed and this generates Majorana masses for the neutrinos. In an underlying Grand Unified theory we expect these terms to be generated by the usual see-saw mechanism.

Of course the critical question is, how the  $\mu$ -term is generated. There are two ways this can happen, either by a  $D$ -term of the form  $X^\dagger H \bar{H}$  [18] or via an  $F$ -term of the form  $Y H \bar{H}$  [20] where  $X, Y$  may be a single field or a composite operator (cf. also the discussion in [21]). The  $\mathbb{Z}_4^R$ -charge of  $X$  and  $Y$  must be 0 and 2 respectively and a  $\mu$ -term is generated if the  $F$ -term of  $X$  or the  $A$  term of  $Y$  acquires a vacuum expectation value (VEV). Both cases break the  $\mathbb{Z}_4^R$  symmetry leaving a  $\mathbb{Z}_2$  symmetry unbroken that, together with invariance of the Lagrangian under a change of sign of the fermion fields, is equivalent to the usual  $R$ -parity of the MSSM. Such a breaking is necessary to allow for gaugino masses. In fact we expect such breaking of the symmetry to occur through non-perturbative effects since the  $\mathbb{Z}_4^R$  is anomalous in the absence of a GS term.

As discussed in the next section, a plausible origin of such non-perturbative terms is through a hidden sector that dynamically generates a vacuum expectation value for the superpotential via a gaugino condensate. This corresponds to identifying  $Y$  with  $\mathcal{W}$ , the latter being the order parameter of  $\mathbb{Z}_4^R$  breaking. In this case the  $\mu$  term is of  $\mathcal{O}(\langle \mathcal{W} \rangle / M^2)$  where  $M$  is the messenger field mass. For the case of gravity mediation (SUGRA)  $M$  is the Planck mass and  $\mathcal{W}$  is also the order parameter for SUGRA;  $\langle \mathcal{W} \rangle / M_P^2 = m_{3/2}$  is the gravitino mass. A specific realization in string theory will be briefly discussed in the next section and in more detail in [15].

There remains the question of the additional SM singlet states that were required to cancel the grav-grav- $\mathbb{Z}_N^R$  graviton anomaly. Their charges are such that  $\sum_i (s_i - 1)$  is odd and so there must be at least one state,  $S$ , with even  $\mathbb{Z}_4^R$  charge. However, the GS mechanism requires the presence of a light axion, and the axino contribution turns out to cancel the grav-grav- $\mathbb{Z}_4^R$  anomaly. The minimal realization of our  $\mathbb{Z}_4^R$  symmetry is therefore a setting in which supersymmetry is broken by the dilaton type multiplet  $S$  containing the axino/dilatino; details will be given elsewhere [15]. It is remarkable that gravitational anomalies lead us to introduce this sector, such that supersymmetry is broken “outside the MSSM”, consistently with phenomenological requirements. That is, the missing spin-1/2 field is needed to give mass to the gravitino. As discussed above, the VEV of the hidden sector superpotential represents an order parameter for  $\mathbb{Z}_4^R$  breaking.

We can now determine the phenomenology of the  $\mathbb{Z}_4^R$  model after supersymmetry breaking. The residual  $\mathbb{Z}_2$  matter parity ensures that the renormalisable terms are identical to those of the usual MSSM with no baryon- or lepton-number violating terms. It also ensures that the

supersymmetric partners of SM states can only be pair produced and that the LSP is stable and a dark matter candidate. The  $\mu$ -term is of the same order as the other visible sector supersymmetry breaking terms and thus the model has completely solved the  $\mu$ -problem. The lowest order baryon- and lepton-number violating terms occur at dimension five, but these operators are strongly suppressed by a non-perturbative factor of  $\mathcal{O}(\langle \mathcal{W} \rangle / M_P^4)$ . For the case of SUGRA this is of  $\mathcal{O}(m_{3/2} / M_P^2)$  and is negligible such that the dimension six proton decay operators will be dominant.

Finally we should consider the cosmological implications of the model. Since it involves a spontaneously broken discrete symmetry one must worry about domain walls being produced in the early universe and dominating the energy density today. A general discussion of walls resulting from the breaking of discrete symmetries has recently appeared [22] (see also [23]) and we refer the reader to it for details appropriate to various choices of the messenger scale. For the case of gravity mediation the domain walls form at the intermediate scale of  $\mathcal{O}(10^{12} \text{ GeV})$ . Provided the Hubble scale during inflation is below this scale, domain walls have sufficient time to form and then they will be inflated away. However to avoid recreating them it is necessary that the reheat temperature after inflation should be less than  $\mathcal{O}(10^{12} \text{ GeV})$ . Given that the gravitino and thermal moduli destabilization [24] bounds require a reheat temperature much below this we conclude that the domain walls in this case do not introduce a significant new problem for SUGRA. Of course one must still deal with the Polonyi problem [25] associated with the energy released if there are light moduli fields but this is not affected by having an underlying  $\mathbb{Z}_4^R$  symmetry.

#### IV. STRING THEORY REALIZATION

Compactified string theories often generate discrete gauge symmetries in the low energy effective Lagrangian so it is appropriate to ask if they can provide the origin of the  $\mathbb{Z}_4^R$ . In particular, (heterotic) orbifolds are known to incorporate discrete  $R$  symmetries in their effective field theory description. These  $R$  symmetries are discrete remnants of the Lorentz group of compact space. Specifically, some of these constructions exhibit a  $\mathbb{Z}_4^R$ , reflecting the discrete rotational symmetry of a  $\mathbb{Z}_2$  orbifold plane  $\mathbb{T}^2 / \mathbb{Z}_2$ .

Making extensive use of the methods to determine the remnant symmetries described in [26], we were able to find examples realizing the  $\mathbb{Z}_4^R$  just introduced, based on the  $\mathbb{Z}_2 \times \mathbb{Z}_2$  orbifold model derived in [27] and similar models, which have three  $\mathbb{T}^2 / \mathbb{Z}_2$  planes. These models have vacua with the exact MSSM spectrum, a large top Yukawa coupling, a non-trivial hidden sector etc. In what follows, we briefly discuss a vacuum exhibiting the  $\mathbb{Z}_4^R$  discussed above, deferring a detailed description to a subsequent publication [15].



We found a configuration in the model [27] in which the  $\mathbb{Z}_4^R$  arises as a mix of the orbifold  $\mathbb{Z}_4^R$  symmetries and other symmetries. The configuration is defined by assigning VEVs to some standard model singlet fields, which break the symmetries at the orbifold point, *i.e.* discrete  $R$  symmetries, discrete symmetries coming from the space group selection rules and the gauged continuous symmetries are broken down to  $G_{\text{SM}} \times \mathbb{Z}_4^R \times \mathbb{Z}_2$ . The VEVs also provide mass terms for the exotics, which are massless at the orbifold point, and allow us to cancel the one-loop Fayet-Iliopoulos term associated with the one anomalous  $U(1)_{\text{anom}}$  of the heterotic orbifold model.

Matter fields, of which we obtain precisely three generations, are identified as fields with  $\mathbb{Z}_4^R$  charge 1. There is one massless Higgs pair (with  $R$  charge 0) at the perturbative level. Unfortunately, the additional  $\mathbb{Z}_2$ , which we cannot break, forbids some Yukawa couplings such that the charged lepton and  $d$ -type Yukawa couplings  $Y_e$  and  $Y_d$  have rank 2.

The presence of the unwanted  $\mathbb{Z}_2$  shows that the vacuum is most likely not fully realistic. Nevertheless our findings imply that the  $\mathbb{Z}_4^R$ , which we have identified solely by bottom-up considerations, can arise in potentially realistic string compactifications, where the symmetry has a clear geometrical interpretation. These models have an exact matter parity, a built-in solution to the  $\mu$ -problem and do not suffer from the dimension five proton decay problems. As they are string-derived (and hence UV complete), we can specify the non-perturbative effects that appear to violate the ‘anomalous’  $\mathbb{Z}_4^R$  in more detail. The (universal) anomalies are canceled by the Green-Schwarz mechanism. That is, the imaginary part of the dilaton  $S$  shifts under the discrete transformations. As a consequence, terms of the form

$$\mathcal{W}_{\text{np}} \supset e^{-8\pi^2 S} (A H \bar{H} + \kappa_{ijkl} Q_i Q_j Q_k L_\ell) , \quad (15)$$

where  $A$  and  $\kappa_{ijkl}$  are constants built of some VEV fields, are  $\mathbb{Z}_4^R$  covariant, *i.e.* have  $R$  charge 2. Such terms can be interpreted as being a consequence of some hidden sector strong dynamics (the model under consideration has a hidden  $SU(3)$ ). Assuming that the scale of supersymmetry breakdown and the expectation value of the superpotential are related to this strong dynamics, we obtain a  $\mu$ -term of the order of the gravitino mass (cf. [28]) and coefficients of the dimension five proton decay operators as small as  $\sim 10^{-15}/M_{\text{P}}$ , *i.e.* well below experimental bounds [29].

## V. SUMMARY

Supersymmetric extensions of the SM promise to eliminate the hierarchy problem. However they also introduce

serious potential problems and to be viable they must evade the  $\mu$ -problem and the problem associated with new baryon- and lepton-number violating terms. This suggests that there should be an additional underlying symmetry capable of controlling these terms.

In this paper we have considered the anomaly free Abelian discrete symmetries that forbid the  $\mu$ -term perturbatively. Remarkably, if one also requires that the symmetry should simply be consistent with  $SO(10)$  unification, there is a unique solution, a  $\mathbb{Z}_4^R$  discrete  $R$ -symmetry. At perturbative order it forbids dimension four and five baryon- and lepton-number violating terms. Being anomalous in the absence of Green-Schwarz terms one may expect the symmetry to be broken non-perturbatively, most likely through a gaugino condensate. At the non-perturbative level, both the  $\mu$ -term and dimension five proton decay operators may arise, while the dimension four operators are still forbidden by a non-anomalous subgroup of  $\mathbb{Z}_4^R$  that is equivalent to matter parity. The magnitude of the dimension five terms is such that the limits on nucleon decay are readily satisfied. Inflation can ensure that the domain walls that are produced when the discrete symmetry is broken are not significant provided a mild upper bound on the inflation scale is satisfied.

Discrete  $R$ -symmetries can result from compactified string models as discrete remnants of the Lorentz group of the compact space. We illustrated this in the context of a semi-realistic orbifold model and showed how it gives rise to the MSSM spectrum below the string scale with a  $\mathbb{Z}_4^R$  discrete  $R$ -symmetry.

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[1] G. R. Farrar and P. Fayet, Phys. Lett. **B76** (1978), 575–579.

[2] S. Dimopoulos and H. Georgi, Nucl. Phys. **B193** (1981),

- 150.
- [3] S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Lett. **B112** (1982), 133.
  - [4] L. M. Krauss and F. Wilczek, Phys. Rev. Lett. **62** (1989) 1221.
  - [5] L. E. Ibáñez and G. G. Ross, Phys. Lett. **B260** (1991), 291–295.
  - [6] T. Banks and M. Dine, Phys. Rev. **D45** (1992), 1424–1427, [hep-th/9109045].
  - [7] L. E. Ibáñez and G. G. Ross, Nucl. Phys. **B368** (1992), 3–37.
  - [8] H. K. Dreiner, C. Luhn, and M. Thormeier, Phys. Rev. **D73** (2006), 075007, [hep-ph/0512163].
  - [9] S. Förste, H. P. Nilles, S. Ramos-Sánchez and P. K. S. Vaudrevange, Phys. Lett. B **693** (2010) 386, [1007.3915].
  - [10] L. E. Ibáñez, Nucl. Phys. **B398** (1993), 301–318, [hep-ph/9210211].
  - [11] M. Dine and M. Graesser, JHEP **01** (2005), 038, [hep-th/0409209].
  - [12] T. Araki et al., Nucl. Phys. **B805** (2008), 124–147, [0805.0207].
  - [13] K. Hamaguchi and N. Maru, Phys. Rev. **D67** (2003), 115003, [hep-ph/0302163].
  - [14] L. J. Hall, Y. Nomura, and A. Pierce, Phys. Lett. **B538** (2002), 359–365, [hep-ph/0204062].
  - [15] H. M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg, and P. K. V. Vaudrevange, (2010), *in preparation*.
  - [16] M. Dine and J. Kehayias, (2009), 0909.1615.
  - [17] K. S. Babu, I. Gogoladze, and K. Wang, Phys. Lett. **B570** (2003), 32–38, [hep-ph/0306003].
  - [18] G. F. Giudice and A. Masiero, Phys. Lett. **B206** (1988), 480–484.
  - [19] K. Kurosawa, N. Maru, and T. Yanagida, Phys. Lett. **B512** (2001), 203–210, [hep-ph/0105136].
  - [20] J. E. Kim and H. P. Nilles, Phys. Lett. **B138** (1984), 150.
  - [21] K. Choi, E. J. Chun, and H. D. Kim, Phys. Rev. **D55** (1997), 7010–7014, [hep-ph/9610504].
  - [22] M. Dine, F. Takahashi, and T. T. Yanagida, JHEP **07** (2010), 003, [1005.3613].
  - [23] S. A. Abel, S. Sarkar and P. L. White, Nucl. Phys. B **454** (1995) 663 [hep-ph/9506359].
  - [24] W. Buchmüller, K. Hamaguchi, O. Lebedev, and M. Ratz, Nucl. Phys. **B699** (2004), 292–308, [hep-th/0404168].
  - [25] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby, and G. G. Ross, Phys. Lett. **B131** (1983), 59.
  - [26] B. Petersen, M. Ratz, and R. Schieren, JHEP **08** (2009), 111, [0907.4049].
  - [27] M. Blaszczyk et al., Phys. Lett. **B683** (2010), 340–348, [0911.4905].
  - [28] F. Brümmer, R. Kappl, M. Ratz, and K. Schmidt-Hoberg, JHEP **04** (2010), 006, [1003.0084].
  - [29] I. Hinchliffe and T. Kaeding, Phys. Rev. **D47** (1993), 279–284.